Appendix I: An Explanation of Flicker-Noise and Noise Processes in Clocks

When Jim and I were characterizing clock noise back in the 1960s, we found that six noise models worked very well in modeling and describing the time deviations we observed. Mathematically, they are easily described as f^0, f^{-1}, f^{-2}, f^{-3}, f^{-4}, and f^{-5} where f is the Fourier frequency behavior of the time deviations. This is like a spectrum of noise colors that cover much of the variabilities we see in nature – not just in clocks. The more one studies these the richer one's observations become, and you will see later, the more these various spectral noise colors are around us and have an effect on much of what we do. As a business example, stock market pricing often behaves in a flicker-noise fashion.

In laymen's terms, an f^0 Fourier spectral density of time variations is like the flip of a coin. If one continued to flip a coin – not a biased Las Vegas one – then this series of flips creates a random and uncorrelated sequence of heads and tails; this is called a white-noise process. This would be the case for an atomic clock if it moved fast a nanosecond (ns, which is a billionth of a second) one day when the coin turned up heads and it moved late a ns if the coin turned up tails. In this thought experiment, whether the clock moves fast or slow a ns depends on the flip of the coin each day. Since the flip of a coin is a random and uncorrelated process from one flip to the next, all Fourier frequencies are equally probable; hence, the spectral density $f^0 = 1$. In other words, one cycle per day variations is as likely as one cycle per week, as one cycle per month, as one cycle per year, etc. This is the kind of noise, which is called white-noise, we see in the short-term time variations of a hydrogen maser atomic clock.

The spectral density f^{-2} is a very important one for atomic clocks and is appropriately called random-walk or drunkard-walk. One can think of it as taking a step forward if the flip of the coin is heads and backwards if the flip of the coin is heads. If you line up a row of people, each having a coin, standing side by side (shoulder-to-shoulder), and you ask them all to flip their coin a 100 times and they walk forward or backward if they flip heads or tails, respectively, you can show statistically that as you look down that row of people you will see what is called a normal or Gaussian distribution of the density of people, and the width of the distribution will be square-root of 100 or 10 steps away from the origin. Every person's position in that distribution is a perfect memory of 100 flips of his or her coin. The width of the distribution is given by the standard deviation of the population – typically designated by σ. If you don't know the definition of standard deviation, Wikipedia has a good one. For a normal distribution, 68.2 % of the people in our line will be within one σ of where they started. Random-walk time deviations are what we see in most traditional atomic clocks like those using cesium, rubidium, and all the optical frequency atomic clocks. I will explain why this is so in laymen's terms.

How the atoms are used in an atomic clock makes a difference in what kind of noise appears in the time deviations. In a hydrogen maser, each atom is in its excited quantum state and is focused into a resonate cavity tuned to the 1.4 GHz clock transition frequency for the hydrogen atom. The preceding atoms have set up an electromagnetic oscillating field, where all the photons given off by them are marching together; we say they are in phase. In laymen's terms, they are in time synchronization. The new atom coming in is stimulated to give off its photon in phase with its neighbors. This synchronization is why it is called a maser (Microwave Amplification by Stimulated Emission of Radiation). Such atomic clocks are called active atomic clocks, because they actively emit a signal from the atoms, and in the ideal, the time or phase deviations are like a flip of the coin. Each atom tends to be synchronized resonant frequency of the electromagnetic signal in the cavity, which is mainly determined by the atoms coming in. There is noise present keeping that from happening perfectly, giving the output a white-noise spectrum in the time deviations.

A LASER is a similar acronym, but in this case it is Light Amplification by Stimulated Emission of Radiation. Lasers are absolutely necessary for optical atomic clocks, for scanning your groceries at the store, and for the policeman checking the speed of your car.

Most atomic clocks operate in the passive mode. By this we mean that a local oscillator is frequency locked to the desired atomic-frequency clock quantum transition. In the ideal scenario, the servo locking this local oscillator will be limited by some level of white-noise as it hunts for this ideal atomic resonance frequency. So for one moment the frequency may be too high and the clock runs too fast during that moment. The next moment the frequency may be too low and the clock runs too slow during that moment. The accumulated time error is a "perfect memory" – like the random-walk process above – of every white-noise frequency deviation of the locked oscillator from the true resonance of the atom. So we see that the spectral density of the time error of a passive ideal atomic clock will go as f^{-2}, while the frequency deviation spectrum will be a white-noise process, f^0.

Jim and I and others observed that the long-term frequency deviations of almost all clocks, atomic or otherwise, have a spectral density of the frequency deviations varying as f^{-1}, and environmental perturbations often induce frequency deviations varying as f^{-2}. We found that the time deviations of the Earth as a clock vary as f^{-3} from about one cycle per day down to one cycle per year. Then for even lower Fourier frequency deviations the Earth varies as f^{-5}, which makes it a very poor clock in the long-term. Its short-term variations are vastly inferior to that of atomic clocks.

So we see that flicker-noise, f^{-1}, has a spectral density that is half way between white-noise, f^0, and random-walk noise, f^{-2}, and we further observe that flicker-noise is not only common in the frequency deviations of atomic clocks, but ubiquitous in nature. Hence, the characterization techniques outlined in this Book can be used in many other areas of metrology. We can use a computer to generate these different noise processes. Dr. Richard Voss generated some music using f^0 noise and it drives you crazy. He then shared with us music generated from an f^{-2}

process and it would put you to sleep. He then, remarkably, generated some music from a flicker-noise, f^{-1} process, and it was pleasing to listen to!

Flicker-noise is sometimes called pink noise or 1/f noise. Other classic examples of its occurrence are in the noise in semiconductor junctions, the voltage across a neuron, the height of the flood stage of the river Nile over the millennia that it has been measured, and there are many, many more examples in nature and in metrology (volt standards, gauge blocks, etc.) in addition to atomic and regular clocks. As an example of neuron flicker noise, if you tried to follow a straight line on a bicycle, your deviations from the straight line would have a spectrum varying as f^{-3}, since your body adds up or integrates your body's neuron noise when steering a bike.

If you average white-noise, it has a well behaved average value. Your confidence on the estimate of its mean will improve as the square root of the number of measurements. As you approach a spectrum of 1/f noise (flicker-noise), you find mathematically that its mean no longer exists; it is unbounded. The statistical tools that we have developed over the years allow us to deal with this unbounded nature and to characterize these different processes in nature. The mathematics of 1/f noise is challenging; it reminds me of agency or free choice when looked at a certain way as I considered its unbounded nature. It is next to being convergent mathematically, but it is not. Flicker-noise processes are self-similar; like humans are similar, but each of us is different. Our choices in life make all the difference.